X11213   Reg. No. :

Question Paper Code : X 11213

B.E./B.Tech. DEGREE EXAMINATIONS, NOV./DEC. 2020
First Semester
Civil Engineering
MA 8151 : ENGINEERING MATHEMATICS – I
(Common to all Branches Except Marine Engineering)
(Regulations 2017)

Time : Three Hours   Maximum : 100 Marks

Answer ALL questions.

PART – A   **(10×2=20 Marks)**

1. If $\lim_{x\to 1}\frac{f(x)-8}{x-1}=10$, then find $\lim_{x\to 1} f(x)$.

2. If $xe^y = x - y$, then find dy/dx by implicit differentiation.

3. Find $\partial u/\partial x$ and $\partial u/\partial y$ when $u(x, y) = x^y + y^x$.

4. If $z = xf\left(\frac{y}{x}\right)$, then find the value of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$, using Euler's theorem.

5. Let A denote the area of the region that lies in the graph of $f(x) = \sqrt{\sin x}$ between 0 and π. Use right endpoints to find an expression for A as a limit. (Do not evaluate the limit).

6. Determine whether integral $\int_1^\infty \frac{\ln(x)}{x}\,dx$ is convergent or divergent. Evaluate it, if it is convergent.

7. Find the area of a circle of radius 'a' by double integration in polar coordinates.

8. Evaluate $\int_{x=0}^{1}\int_{y=0}^{2}\int_{z-1}^{2} xy\,dx\,dy\,dz$.

9. Find the particular integral of y'' + 2y' + y = cosh x.

10. Solve y''' + 2y'' + y' = 0.

PART – B **(5×16=80 Marks)**

11. a) i) Use the intermediate value theorem to show that there is a root of the equation $\sqrt[3]{x} = 1 - x$ in the interval (0, 1). **(6)**

ii) Show that the function $f(x) = |x - 6|$ is not differentiable at 6. Find a formula for first derivative of f and sketch its graph. **(10)**

(OR)

b) i) Find the equation of the tangent line to the curve $y = x^4 + 2x^2 - x$ at the point (1, 2). **(4)**

ii) Find the local maximum value, local minimum value, the interval of concavity and the inflection points of a function $f(x) = x^3 - 3x^2 - 12x$. Also sketch the graph of f that satisfies all the above conditions. **(12)**

12. a) i) Let $u = 3x + 2y - z$, $v = x - 2y + z$ and $w = x(x + 2y - z)$. Are u, v and w functionally related ? If so, find this relationship. **(8)**

ii) Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface area 432 sq.cm. **(8)**

(OR)

b) i) If $z = f(x, y)$, where $x = e^u + e^{-v}$ and $y = e^{-u} - e^v$, then show that

$$x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} - \frac{\partial z}{\partial v}.$$ **(8)**

ii) Find the Taylor's series expansion of $f(x, y) = x^2 y^2 + 2x^2 y + 3x y^2$ in powers of $(x + 2)$ and $(y - 1)$ up to the second degree terms. **(8)**

13. a) i) Prove that $\int_{-\pi}^{\pi} \sin mx \sin nx \, dx = \begin{cases} 0 & \text{if} \quad m \neq n \\ \pi & \text{if} \quad m = n \end{cases}$, where m and n are positive integers. **(8)**

ii) Evaluate the integral $\int \frac{x^2 - 2x - 1}{(x-1)^2(x^2+1)} dx$. **(8)**

(OR)

b) i) Evaluate the integrals 1) $\int x^3\sqrt{x^2+1}\, dx$ and 2) $\int_0^1 \frac{1}{\left(1+\sqrt{x}\right)^4} dx$. **(8)**

ii) Find the values of p for which the integral $\int_0^1 x^p \ln(x)\, dx$ converges, and evaluate the integral for those values of p. **(8)**

14. a) i) Change the order of integration in $\int_0^1 \int_y^{2-y} xy\, dx\, dy$ and then evaluate it. **(8)**

ii) Evaluate $\iiint_V x\,y\,z\,dx\,dy\,dz$, where V is the volume of the positive octant of the sphere $x^2 + y^2 + z^2 = 1$ by transforming to spherical polar coordinates. **(8)**

(OR)

b) i) Evaluate $\iint_D xy\sqrt{(1-x-y)}\,dx\,dy,$ where D is the region bounded by $x = 0$, $y = 0$ and $x + y = 1$, using the transformation $x + y = u$, $y = uv$. **(8)**

ii) Find the volume of the cylinder bounded by $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$, using triple integral. **(8)**

15. a) i) Solve the simultaneous differential equations
$\frac{dx}{dt} + 2y + \sin t = 0, \frac{dy}{dt} - 2x - \cos t = 0$, given that $x = 0$, $y = 1$ when $t = 0$. **(8)**

ii) Use the method of undetermined coefficients to solve

$y'' - 5y' + 6y = e^{3x} + \sin x$. **(8)**

(OR)

b) i) Solve $x^2 y'' + xy' + y = x \ln(x)$. **(8)**

ii) Using the method of variation of parameters, solve $y'' + y = x \cos x$. **(8)**
